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DYNAMIC VISCOSITY AND MATERIAL RELAXATION TIME DURING SHOCK LOADING

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The dynamic viscosity and characteristic relaxation time at various scale-structural levels of deformation of a shock-loaded medium are determined using concepts of multilevel solid-state mechanics. The notion of the quasi-time fractal dimension is introduced and used to calculate the indicated characteristics. Computational-experimental data for the viscosity and relaxation time are given for three materials: M2 *copper,* AMg6 *aluminum base alloy and Armco iron.*

Key words: *dynamic viscosity, relaxation, structural level, fractal dimension.*

Introduction. In the rapid plastic deformation of metals under shock loading, an important factor is the dissipative energy loss due to the dynamic viscosity, which is generally the material response to the rate of the process.

The viscous resistance of a material is characterized by the proportionality coefficient at the strain rate tensor in the equations for strain resistance or for the dissipative function, which is called the dynamic viscosity μ. Generally, this quantity is a fourth-rank tensor.

A considerable body of data exists on the dynamic viscosity of many metals and alloys over a wide range of strain rate ($\dot{\varepsilon} = 10^3$–10^6 sec^{-1}) (see [1] and the bibliography therein). However, first, these data differ by several orders of magnitude (even for the same materials and the same loading parameters), which is apparently due to the use of different methods to determine the dynamic viscosity and its dependence on the structural level scale at which it is obtained [1]. Second, the dynamic viscosity is determined, as a rule, in the same experiments in which it is used for modeling and prediction.

The material characteristic which is inversely proportional to the dynamic viscosity is the shear stress relaxation time. It should be noted that methods for direct experimental determination of this characteristic under shock loading conditions are not currently available. It is therefore, reasonable to develop a theoretical model (at least, semi-empirical) to calculate the dynamic viscosity or characteristic relaxation time through fundamental structural microconstants of material and (or) the minimum number of experimental parameters at the lowest (highest) structural level. For other scale-structural levels of deformation (which will be considered below), it is reasonable to determine the viscosity and relaxation characteristics (these parameters play an important role in modeling multilevel plastic deformation) through the characteristic time or geometrical scale of the process.

As is known, crystalline materials are fading-memory materials which can be described using a generalized Maxwell relaxation model [2]. Depending on the material parameters included in this generalized model, it is possible to obtain various particular governing equations describing the deformation resistance of elastoplastic, viscoplastic, and relaxing media [3].

As noted above, the dynamic viscosity can be defined in terms of the characteristic relaxation time t_{r}, which generally depends on the stress state of the medium and its temperature T [2]. For a generalized Maxwell medium, the relation between μ and t_{r} has the form [3]

$$\mu = 2Gt_{\mathrm{r}}(\sigma_i, T), \tag{1}$$

where G is the shear modulus and σ_i is the main stress.

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In the case of a viscoelastic body, it has been proposed [4] to define the dynamic viscosity by the relation

$$\mu = Gt_{\rm r}(\sigma_i, T),$$

where $t_{\rm r}$ is the time during which the initial stress decreases by a factor of $e \approx 2.71$.

According to modern concepts of the mechanics and physics of deformable solids, plastic deformation and fracture occur at several structural levels with different physical processes, activation energies, characteristic scales, and characteristic stress relaxation time [5].

At each level, the characteristic relaxation time is determined by the viscous damping time of strain carriers — crystal structure defects of the given scale-structural level.

According to the existing classification of scale-structural levels, deformation and fracture occur at several levels: the microscopic (atomic) level, i.e., the level of point defects, single dislocations, grains, and their simplest combinations (the characteristic scale of this level L_1 is approximately equal to several interatomic distances), the macroscopic level, i.e., the level with a characteristic size $L_4 = 10^{-3}$–10^{-2} m, and the mesoscopic (intermediate) level with several characteristic sizes. The mesolevel I includes the level of dislocation clusters of size $L_{\rm I2}$ from 1 nm to 2 μm and the subgrain fragmentary level with $L_{\rm I3} = 2$–20 μm; the mesolevel II include the grain level (the characteristic size $L_{\rm II4}$ is equal to the average grain size) and the so-called superstructural level ($L_{\rm II5} = (2$–$10)d_{\rm g}$) [6].

1. Fractal Model of Relaxation. It has been shown [1] that

$$\mu = 2G\sum_{i=1}^{n}\mu_i, \tag{2}$$

where μ_i is the dynamic viscosity of the ith scale-structural level; the summation is performed over n levels which of significance in plastic deformation.

In view of (1) and (2), the expression for the quantity μ can be written as

$$\mu = 2G\sum_{i=1}^{n}t_i. \tag{3}$$

In turn, the characteristic scales of the relaxation time t_i are defined by the corresponding characteristic scales of the structural deformation levels. To determine t_i, we divide the deformation region into one-dimensional cells of size $\Delta\varepsilon$. By decreasing or increasing this size, it is possible to obtain different number of cells. The principle of scale invariance of plastic deformation [7] (for high-velocity deformation, this principle was validated in [6]) allows one to establish the relationship between the number of cells N and their size using the basic relation of fractal analysis

$$N(\Delta\varepsilon) \sim \Delta\varepsilon^{-D}, \tag{4}$$

where the exponential parameter D is the fractal dimension and the characteristic size $\Delta\varepsilon$ is the minimum distinguished scale of the deformation region.

We assume a discrete spectrum of internal successive (Poisson flow of events) relaxation dissipative processes in this deformation system. As is known, for a Poisson process, the time can be represented in the form of the fractal set of times of events [8], i.e., in the case considered, the characteristic relaxation time should satisfy the relation

$$t_{\rm r}(\Delta\varepsilon) \sim \Delta t^{-D_t},$$

where Δt is the chosen time scale and D_t is the fractional exponent of the degree, which is the time fractal dimension.

Because the scale Δt can be represented in the form $\Delta t = \Delta\varepsilon/V$ (V is the relaxation velocity), the expression $t_{\rm r}(\Delta\varepsilon) \sim \Delta t^{-D_t}$ can be written similarly to (4) as

$$t_{\rm r}(\Delta\varepsilon) \sim \Delta\varepsilon^{-D_t}. \tag{5}$$

Here D_t can be called the quasi-time fractal dimension because, although this parameter defines time, the base of the exponential function (5) is a parameter with the spatial dimension.

Given the time $t_{\rm r}$ of the entire relaxation process, by logarithming from relation (5), it is possible to determine the quasi-time fractal dimension D_t (letting $\Delta\varepsilon$ go to zero):

$$D_t = -\lim_{\Delta\varepsilon\to 0}\frac{\log t_{\rm r}}{\log\Delta\varepsilon} \tag{6}$$

(the logarithmic base is arbitrary). Determining the quasi-time dimension in this manner, as $\Delta\varepsilon \to 0$ it is reasonable to choose the average distance between two equilibrium positions of the atom $\delta \approx 10^{-10}$ m [4].

It is more difficult to determine the time $t_{\rm r}$. In [2] and other earlier papers of the same author, it is noted that, in high-velocity dynamic processes for metals, the shear stress relaxation time is $t_{\rm r} = 10^{-6}$–10^{-5} sec. In this case, however, at strain rates $\dot\varepsilon = 10^5$–10^6 sec^{-1}, the viscous component of the strain resistance $\sigma_g \approx \mu\dot\varepsilon$ for any metals and alloys is equal to $2G$ (or G, according to Frenkel), which is not observed in experiments. (Dynamic yield points exceed static values by a factor of 3–4, and in some cases by a factor of 6 (for example, for aluminum) and a factor of 9 (for magnesium) [9], but this is an exception rather than a rule, because a paper [9] also gives data on a decrease in the yield point under shock loading. A similar tendency toward a sharp decrease in the yield point was noted in [10]).

Apparently, the most reliable data on the relaxation time of the entire process should be considered the values of $t_{\rm r}$ obtained from formula (1) by substituting the dynamic viscosity obtained in the experiments described in [11] (see also [12]). From these experiments, it follows that for simple steels (St. 6, St. 20, and St. 45), $t_{\rm r} = 0.25$ μsec for $\dot\varepsilon = 6 \cdot 10^3$ sec^{-1} and $t_{\rm r} \approx 0.0135$ μsec for $\dot\varepsilon = 10^4$–$5 \cdot 10^4$ sec^{-1}. For D16 aluminum alloy $t_{\rm r} = 0.6$ μsec for $\dot\varepsilon = 6 \cdot 10^3$ sec^{-1} and $t_{\rm r} \approx 0.03$ μsec $\dot\varepsilon = 10^4$–$5 \cdot 10^4$ sec^{-1}.

After the determination of the quasi-time fractal dimension, we can write a more accurate relation for the characteristic total stress relaxation time at each level:

$$t_{{\rm r},i}(L_i) = K_i(L_i, D_t)\Delta\varepsilon^{-D_t}. \tag{7}$$

Here $K_i(L_i, D_t)$ is a coefficient which characterizes the structural scale of physical processes and matches dimensions.

At this structural level, the characteristic relaxation time t_i is equal to

$$t_i = t_{{\rm r},i+1} - t_{{\rm r},i}. \tag{8}$$

To determine the coefficient $K_i(L_i, D_t)$ at each scale-structural level, as a first approximation we assume that it is a linear function dependent on the scale.

The coefficient K_1 for the microscopic and macroscopic levels is the easiest to determine. Because the characteristic total relaxation time for the microscopic level can be determined from the relation [4] (a paper [4] also gives other relations for $t_{{\rm r},1}$)

$$t_{{\rm r},1} = \alpha B/(2Gb^2N_m), \tag{9}$$

where $\alpha \approx 3$ is a coefficient that takes into account the multiplication of slip systems and various crystal orientations in a polycrystal [13], B is the coefficient of viscous dislocation damping, and N_m is the mobile dislocation density,

$$K_1 = \alpha B\Delta\varepsilon^{D_t}/(2Gb^2N_m). \tag{10}$$

Thus, key parameters determining the relaxation process are the viscous damping coefficient B and the mobile dislocation density N_m.

Assuming that, at high loading rates, the phonon viscosity makes a major contribution to the dislocation damping, the coefficient B can be determined from the approximate relation [13] (the formula underestimates the values of B by approximately an order of magnitude but allows one to determine the required characteristics using material microconstants)

$$B = 3bkT/(10a^3C_2), \tag{11}$$

where k is Boltzmann's constant, T is the temperature, a^3 is the volume of the atomic cell, and C_2 is the transverse-wave velocity.

In the initial stage of plastic deformation (described by the dislocation plasticity equations) under shock-loading conditions with two mechanisms of dislocation multiplication (regenerative and heterogeneous), the mobile dislocation density is given by the relation [14]

$$N_m = (N_{m0} + \alpha_1\gamma)\exp(H_1\dot\gamma/\tau), \tag{12}$$

where N_{m0} is the initial mobile dislocation density, α_1 is the dislocation multiplication coefficient, γ and $\dot\gamma$ are the shear strain and its rate, H_1 is a constant, and τ is the shear stress.

An analysis of formulas (7) and (10)–(12) shows that the characteristic relaxation time increases with increasing structural scale of deformation, temperature and shear strain and rate and with decreasing shear stress.

TABLE 1

Material Characteristics and Initial Data for Calculation $B = 10^{-4}$ Pa · sec and $N_m = 10^{13}$ m^{-2}

Material	G, GPa	σ_b, MPa	ρ, tons/m^3	b, nm	Δu_{10}, m/sec	Δu_{50}, m/sec	$\dot{\varepsilon} \cdot 10^{-5}$, sec^{-1}
M2	46.0	220	8.9	0.240	22.6	49.4	2.060
AMg6	27.7	370	2.7	0.286	17.0	30.4	0.190
Armco iron	81.4	365	7.8	0.250	19.6	22.3	0.873

Note. Data on Δu_{10} and Δu_{50} for Armco iron were provided by Yu. I. Meshcheryakov (Institute of Problems of Machine Science, Russian Academy of Sciences, St. Petersburg).

Let us estimate the order of magnitude of the coefficient K_1, for example for St. 20 steel. We assume that the parameters included in (10) have the following orders of magnitude: $B \approx 10^{-4}$ Pa· sec, $N_m \approx 10^{13}$ m^{-2}, $G \approx 8 \cdot 10^{10}$ Pa, $b = 2.5 \cdot 10^{-10}$ m, and $\Delta\varepsilon_1 \approx 10^{-10}$ m. For $\dot{\varepsilon} = 10^4$–$5 \cdot 10^4$ sec^{-1}, the quasi-time fractal dimension for St. 20 steel determined from formula (6) is $D_t = -0.79$. Substitution of these values into formula (10) yields $K_1 \approx 2.4 \cdot 10^{-2}$.

For the macroscopic level, we have $K_4 = 1$.

Substituting more accurate experimental-computational values of the parameters into the relations obtained, it is possible to obtain fairly realistic values of the characteristic relaxation times and dynamic viscosity. This leads to the problem of determining the relaxation time from relation (9) because, in this case, it is necessary to find the mobile dislocation density N_m. Mobile dislocations make a certain contribution to the total number of all dislocations N_n. It can be assumed that under shock loading, the dislocation density N reaches saturation at the level of the limiting values $N_n = 10^{14}$–10^{15} m^{-2}. In this case, the value $N_m = 10^{13}$–$5 \cdot 10^{13}$ m^{-2} (the limiting mobile dislocation density calculated in [15] is $3 \cdot 10^{13}$ m^{-2}, a paper [16] gives a value $N_m \approx 10^{13}$ m^{-2} for the case of strong shock waves for aluminum, copper, and steel).

2. Computational-Experimental Validation of the Model. To estimate the adequacy of the proposed model using computational-experimental methods, we determined the dynamic viscosity (and, hence, the characteristic relaxation times) for three materials: M2 copper, AMg6 aluminum base alloy, and Armco iron (initial data are given in Table 1).

The characteristics for the macrostructural level in copper and aluminum alloy were determined using the Hopkinson split bar technique (the key parameters are given in [17]), and the data for Armco iron are taken from [12]. Once the dynamic breaking strength $\sigma_{\text{b.d.}}$ is determined, the dynamic viscosity can be found from the relation

$$\mu_4 = (\sigma_{\text{b.d.}} - \sigma_0)/(3\dot{\varepsilon}),$$

where σ_0 is the bending strength and $\dot{\varepsilon} = 5 \cdot 10^3$ sec^{-1} is the strain rate determined from the Hopkinson split bar. The characteristic relaxation time was given by relation (1).

By using the dynamic viscosity relations obtained in [1]

$$\mu \approx \rho(\Delta u_i)^3/(\varepsilon_i \dot{\varepsilon}^2) \tag{13}$$

(ρ is the density of the sample material and Δu_i is the scatter of velocities at the ith scale level), we determined the dynamic viscosity and relaxation time at the mesolevels I and II.

The scatter of particle velocities Δu_i was determined using a high-velocity laser interferometer [1, 18]. This technique allows one to determine the scatter of velocities of plastic-strain carriers at two scale levels. The first level corresponds to the motion of carriers whose size is much smaller than the diameter of the laser beam focused onto the free surface of the target. In the experiments, the beam diameter was 50 μm. The lower limit was determined by the geometrical optics laws, according to which a sharply defined beam reflection can be obtained if the sizes of surface heterogeneities are much larger than the laser radiation wavelength (in this case, 0.6 μm). Thus, the size of the structural element (and, hence, the characteristic scale) is in the range $0.6\ \mu\text{m} \ll L_i \ll 50\ \mu\text{m}$. Using the normal size distribution in this range, it is possible to estimate the average size of the structural element $L_i = (L_{\max} - L_{\min})/\beta$, where β is a coefficient whose values are given in [19] (in this case, $\beta = 5$). As a result, the average characteristic size of the structural element is $L_i \approx 10$ μm. This estimate also corresponds to the heterogeneities (shear and rotational elements) observed using light and electron scanning microscopy. Figure 1

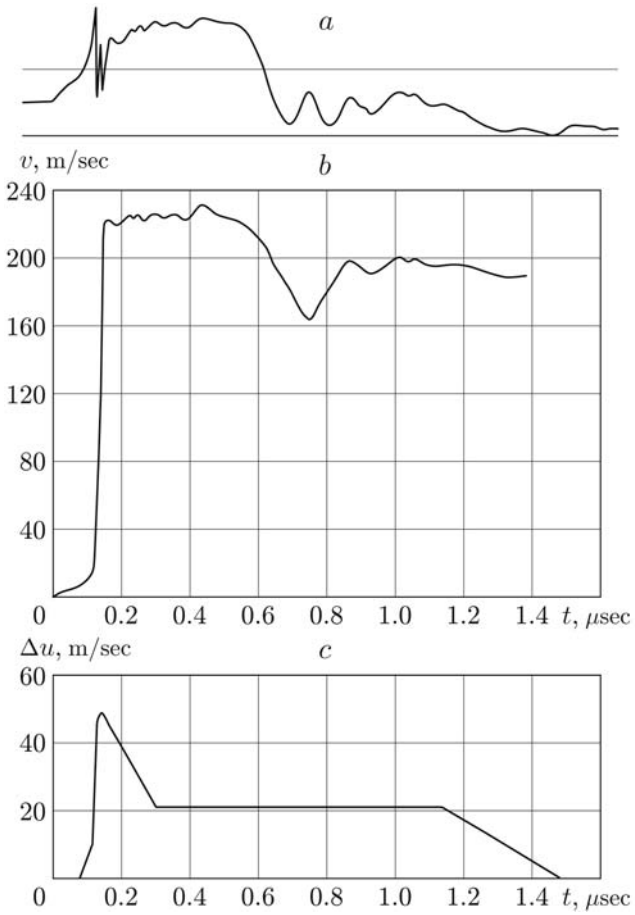

Fig. 1. Fringe pattern (a), curve of the velocity of the free surface versus time (b), and scatter of velocities (c) for an M2 copper sample at an impact velocity of 261 m/sec.

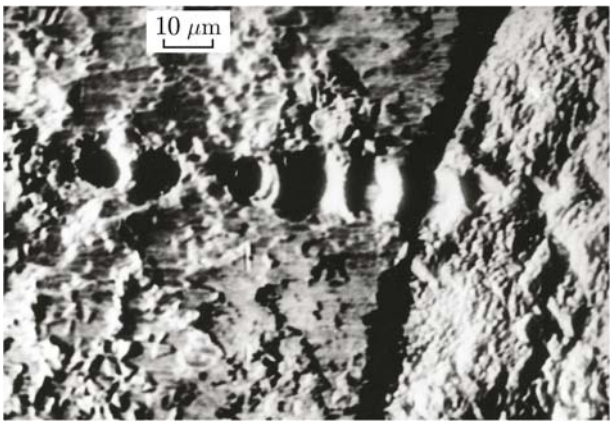

Fig. 2. Fracture pattern of rotational cells formed in an M2 copper sample upon impact.

TABLE 2

Viscous and Relaxation Characteristics of Metals

Material	t_1, nsec	μ_1, Pa · sec	t_{I2}, nsec	μ_{I2}, Pa · sec	t_{g3}, nsec	μ_{g3}, Pa · sec	t_{II4}, nsec	μ_{II4}, Pa · sec
M2	5.66	521	5.67	522	5.76	530	5.93	546
AMg6	6,60	365	6.70	370	7.00	387	7.80	431
Armco iron	2.95	480	2.97	483	2.98	485	3.00	488

Material	t_{II5}, nsec	μ_{II5}, Pa · sec	t_4, nsec	μ_4, Pa · sec	t_1, nsec	μ_1, Pa · sec	μ_{10}, Pa · sec	μ_{50}, Pa · sec
M2	7.89	726	100	10^4	1.75–4.00	165–370 [16, 20]	490	505
AMg6	16.30	901	135	$7.5 \cdot 10^3$	1.4–15.0*	75–800* [16, 21]	385	440
Armco iron	3.20	529	15.3	$2.5 \cdot 10^3$	2.2–3.7	360–600 [10, 22]	702	216

Notes. 1) For M2 copper, AMg6 aluminum base alloy, and Armco iron, calculations were performed at $D_t = -0.73$, -0.67, and -0.81, respectively.
2) In the calculations, the characteristics scales for levels I2, I3, II4, and II5 were set equal to 2, 20, 60, and 500 μm, respectively.
3) Data for technically pure aluminum and 6061-T6 aluminum base alloy are marked by an asterisk.

gives a fringe pattern, a curve of the velocity of the free surface versus time, and the scatter of velocities for an M2 copper sample at impact velocities 261 m/sec. Figure 2 gives a fracture pattern of rotational cells formed in an M2 copper sample upon impact.

The second scale level at which the time characteristic of the dynamic deformation process can be recorded corresponds to the rigid-body motion of the entire surface element probed by the laser beam and this motion is stochastic, i.e., the velocities of neighboring sites of the free surface also have a certain scatter. This conclusion is confirmed by fringe patterns, which show a decrease in the number of beats of the interference signal compared to the number of beats corresponding to the average macroscopic velocity [18]. The viscosity and relaxation time characteristics determined using the technique described above are given in Table 2. As noted above, in the experiments, the laser beam was focused to a diameter of 50 μm. Table 2 give values of the viscosity parameters μ_{10} and μ_{50} for the levels with characteristic scales equal to 10 and 50 μm.

A comparison of the obtained results with the experimental-calculated data given in [10, 16, 20–22] (see Table 2) shows satisfactory agreement [taking into account that formula (13) is approximate] among the calculated characteristics obtained for the proposed model (it should be noted that the author of the present paper is unaware of the structural parameters of materials for which data of [10, 16, 20–22]) were obtained.

3. Conclusions. A stress relaxation model using the quasi-time fractal dimension was proposed to determine the dynamic viscosity and the characteristic spectrum of relaxation times.

The introduction of the quasi-time dimensions considerably simplifies the determination of the above-mentioned characteristics for intermediate structural-scale deformation levels in the development and application of multilevel models of deformation.

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REFERENCES

1. G. G. Savenkov and Yu. I. Meshcheryakov “Structural viscosity of solids,” *Combust., Expl., Shock Waves*, **38**, No. 3, 352–357 (2002).

2. S. K. Godunov, *Elements of Continuum Mechanics* [in Russian], Nauka, Moscow (1978).
3. L. V. Al'tshuler and B. S. Chekin, "Structure of shock waves and fundamental equations for metals," *J. Appl. Mech. Tech. Phys.*, No. 6, 910–918 (1987).
4. Ya. I. Frenkel', *Introduction to the Theory of Metals* [in Russian], Nauka, Leningrad (1986).
5. V. I. Vladimirov, V. N. Ivanov, and N. D. Priemskii, "Mesoscopic level of plastic deformation," in: *Physics of Strength and Plasticity* [in Russian], Nauka, Leningrad (1986), pp. 69–80.
6. B. G. Barakhtin and G. G. Savenkov, "Rotational hardening and deformations of steel in the mechanical field of a high-velocity impactor," in: *Memory Shape Effects and Other Promising Materials* [in Russian], St. Petersburg State Univ., St. Petersburg (2001), pp. 326–332.
7. V. E. Panin, "Physical Mesomechanics of Materials," *Izv. Ross. Akad. Nauk, Mekh. Tverd. Tela*, **5**, 88–108 (1999).
8. A. Blumen, J. Klafter, and G. Zumofen, in: L. Pietronero and E. Tosatti (eds.), *Fractals in Physics*, North-Holland, Amsterdam (1986).
9. B. L. Glushak, and S. A. Novikov, and Yu. V. Bat'kov, "Constitutive equation describing high-strain rates of Al and Mg in a shock wave," *Combust., Expl., Shock Waves*, No. 1, 79–83 (1992).
10. J. W. Taylor, "Dislocation dynamics and dynamic yielding," *J. Appl. Phys.*, **36**, No. 10, 3145–3150 (1965).
11. S. M. Popov, "Absolute viscosity of steel," *Inzh. Sb.*, **1**, No. 1, 27–36 (1941).
12. G. V. Stepanov, *Elastoplastic Deformation and Fracture of Materials under Pulse Loading* [in Russian], Naukova Dumka, Kiev (1991).
13. R. J. Clifton, "Dynamic plasticity," *J. Appl. Mech.*, **50**, No. 4b, 941 (1983).
14. Yu. I. Meshcheryakov and G. G. Savenkov, "Oscillations of the plastic wave under high-rate loading," *J. Appl. Mech. Tech. Phys.*, **42**, No. 6, 1023–1028 (2001).
15. G. F. Sarafanov, "Correlation effects in an ensemble of boundary-value dislocations," *Fiz. Tverd. Tela*, **50**, No. 10, 1793–1799 (2008).
16. L. C. Chhabildas and J. R. Asay, "Rise-rime measurements of shock transitions in aluminum, copper, and steel," *J. Appl. Phys.*, **50**, No. 4, 934–941 (1979).
17. G. G. Savenkov, "Critical stresses in spalling and dynamic rupture of metals," *J. Appl. Mech. Tech. Phys.*, **46**, No. 6, 857–860 (2005).
18. Yu. I. Meshcheryakov and A. K. Divakov, "Interference method of recording high-velocity inhomogeneity of particles in elastoplastic loading waves in solids," Preprint No. 25, Leningrad Department of the Institute of Machine Science, Leningrad (1989).
19. D. Hudson, *Statistics*, Geneva (1964).
20. L. V. Al'tshuler and B. S. Chekin, "Rheology of wave deformation in metals," *Combust., Expl., Shock Waves*, No. 5, 660–662 (1983).
21. V. A. Ogorodnikov, A. A. Sadovoi, E. S. Tyun'kin, and N. M. Chulkov, "Viscosity of aluminum and lead in shockwave experiments," *J. Appl. Mech. Tech. Phys.*, **36**, No. 1, 4–9 (1995).
22. G. S. Pisarenko and A. E. Krasovskii, "On the physical theory of the dynamic yield point of crystal materials," *Probl. Prochn.*, No. 11, 11–14 (1970).